

Estimates of the thickness of hydrodynamic and diffusive boundary layers on sucrose crystals under low-frequency mechanical vibrations

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Resume. The paper includes numerical estimates of the thickness of hydrodynamic and diffusive boundary layers on sucrose crystals under low-frequency mechanical vibrations in massecuites of various levels of purity. Conducted analysis allows explaining the intensified effects of sugar massecuites of C-type.

Keywords: low-frequency mechanical oscillations, sugar crystals, massecuites, boundary layers, mechanical vibrations, crystallization.

1. INTRODUCTION

Active and purposeful use of vibrational effects as a controlling factor is very perspective direction of vibrational technology connected with processing of the disperse systems of viscous-suspension type. It is of great significance to investigate the possible forms of relative motion and dynamic balance of separate phases in multiphase medium under periodic influences such as low-frequency vibrations.

In many cases the vibration does not change the essence of technological processes, but it executes a role of peculiar catalyst considerably increasing process productivity, intensity, or product yield. These processes are performed by means of other driving forces of vibrationless nature: temperature gradients, gravitational and molecular interactions, chemical reactions and others. Vibration field imposing onto such processes permits strengthening or relaxing of vibrationless forces and by that makes it possible to control the technological processes. In such cases the selectivity and the directivity of vibration-field action create opportunities for new highly effective technologies and processes [1].

In spite of the fact that nowadays the acoustic methods of influence on food disperse systems are being spread wider and wider in various branches of industries, the selection of conditions of acoustic effects on processes are nevertheless based on an empirical approach. Such approach to application of the acoustic effects on chemical-technology processes, which are distinguished by a significant complexity and frequently represented by the combination of chemical, hydrodynamical, diffusional, and thermal processes, does not promote a correct choice of technological conditions, gives no possibility to optimize them and, therefore, to increase the economic efficiency of processes [2].

2. EXPERIMENTAL PROCEDURE

Recently the interest in researches devoted to the vibrational methods of intensification of heat and mass transfer processes in beet-sugar production continues to grow. Positive influence of low- frequency acoustic field on crystallizing sugar massecuites as well is being substantiated. And the stated considerations about the mechanism of oscillation influence on the mass transfer are based on hypotheses of the effect on a boundary layer and do not contain quantitative relations.

At the same time, die question of estimation of an existing sugar-crystal boundary layer is a matter of principle, because it allows to determine the efficiency of interactions between phases. Conformably to saccharose they more often operate with the general concept of boundary layer without considering die hydrodynamic and diffusional layers' relationship. Such approach is not correct enough.

A laboratory batch vacuum pan crystallizer with vibrostirring mechanism [3] shown in Fig. 1 was used for carrying out the researches on isobaric vacuum massecuite boiling with acoustic effects on the crystallizing slurry.

The apparatus involved a cylindrical vacuum-pan case [1] of 0.22m diameter, supplied with an ellipsoidal bottom [3] having four boiling tubes [4] of 0.029m diameter with electric heaters of 1200 W total power [5]. The vacuum pan is supplied with a condensator [2]. The branch pipe [6] served for initial filling of the vacuum pan with a starting product (9-10 dm³ initial charge's volume) and for feeding during boiling. The central channel [7] of 0.06 m diameter provided the descending natural circulation of the massecuite which probes were taken out through a sampler [8]. A vibrostirring element was represented by a disk [9] of 0.076 m diameter punched by conoidal channels, with narrow holes downwards. The channels was of 90° taper angle and of 0.005 m smaller diameter, the disk was of 0.005 m smaller diameter, the disk was of 0.005 m thickness and of 7.8% free cross-sectional area. The disk [9] was fastened to a rod [10], which had a hermetic outlet from the vacuum pan by means of bellows [11].

Vibrostirring of crystallizing solutions by hermonic oscillations was put into practice by an electromechanical vibroexciter constructed with an electric drive [12], which was a thyristor DC apparatus with smooth rotation frequency regulation of an electric motor [14] within the limits of 60-3000 rpm. The output shaft of the electric motor was connected by means of elastic coupling [15] with the shaft [16] of an eccentric mechanism transforming its rotary motion to reciprocating one of the rod [10]. A driving faceplate [17] and a driven one [18] were used as a crank. The radial displacement of the driven faceplate permitted to set the eccentricity value (oscillation amplitude) from 0 up to 0.007 m. The driven faceplate had a pin [19] with a rolling bearing [20]. Electric motor parameters by current and voltage being monitored with measuring devices [13].

The parameters of vibration were evaluated by measurement of oscillation frequency and amplitude. The frequency above 4.2 Hz was determined with the relative error of 0.5%, lighting up the vibromechanism with a pulsing lamp [21] of a stroboscopic tachometer [22] combined with a frequencymeter [23].

Before swithcing on the vibratory drive, the amplitude of oscillations was measured from the shaft [16] and pin [19] eccentricity within the accuracy of 0.1 mm, making use of vernier callipers. Practically, the eccentricity measurement was executed by measurement of the displacement of faceplates [17] and [18] relatively to each other in a radial direction.

A crystallizing solution temperature being measured with a mercury thermometer [35] accurate within 0.5°C and the temperature of secondary steam was measured with a resistance copper thermometer [36], which was connected to an automatic bridge with diagram-tape temperature recording within the range of 0-100°C to the accuracy of 0.5°C.

The effective viscosity of crystal slurry was monitored by a measuring converter [26] connected to a vibrational low-frequency viscosimeter [27] with a digital display device [28].

The mass fraction of solids in solution was determined with a refractometer [38] after the crystalline- phase recovery in a centrifuge [37].

In the researches carried out, the batch boiling of high- and low-grade sugar massecuites (A- and C-types) of diferent purity under the vibrostirring by infrasonic harmonic oscillations (the frequency $f = 10-12.5$ Hz) with the amplitude $A = 3$ mm was studied. The estimation of the vibration effect was performed with the power of oscillations, initiated in massecuites by the vibratory element, on the bassis of the vibrostirring intensity $J = A^2 f^3$ (Table 1).

3. RESULTS AND DISCUSSION

The obtained experimental results have allowed to calculate the thickness of hydrodynamic and diffusional boundary layers for saccharose crystals. The calculations have an approximate changes of hydrodynamic and mass transfer parameters in the experiments on isobaric vacuum pan boiling of high- and low-grade sugar massecuites with vibrostirring character, because the estimates of an order of the derivatives included in the convective-diffusion equation and the Navier-Stokes one are used.

The equations of liquid movement in a boudary layer admit essential simplification, and the flow may be considered to be flat on a reasonably small part of a crystal because of small boundary-layer thickness.

On the basis of consideration of the convective-diffusion equation for the steady-state mass transfer process in two-dimensional liquid flow the following expression /5/ has been obtained:

$$v_x \cdot C / \Phi d_k + v_y \cdot C / \delta_y \approx D \left[C / (\Phi d_k)^2 + C / \delta_d^2 \right] \quad (1)$$

where v_x and v_y are the components of a liquid flow velocity vector on coordinates x and y along the crystal surface and perpendicularly to it, m/s; C is the relative mass of saccharose in the solution, kg/(kgH₂O); Φ is the shape factor of saccharose crystals ($\Phi \sim 0.704$); d_k is the crystal size, m; δ_d is the diffusional boundary-layer thickness, m; D is the molecular-diffusion coefficient, m²/s.

Table-1. Change in the hydrodynamic and mass transfer parameters during boiling of sugar massecuites combined with mixing induced by mechanical vibrations

Time, s	Temperature, K	Relative mass of		Parameters of oscillations		Diffusion coefficient, $D \cdot 10^{10}$, m ² /s	Local oscillatory Reynolds number Re_0
		solids in solution	saccharose in solution	frequency, Hz	amplitude $A \cdot 10^3$, m		
0	334.0	2.37	2.36	12.50	3	-	0
6000	338.0	3.26	3.24	12.50	3	-	1.31
7200	337.0	3.34	3.32	12.50	3	-	1.10
8400	335.0	3.37	3.35	12.50	3	2.075	0.93
9600	334.0	3.37	3.35	12.50	3	1.993	0.87
0	336.5	2.11	2.10	12.50	3	-	0
5100	340.0	3.58	3.56	12.50	3	2.412	0.97
6300	341.5	3.60	3.58	12.50	3	2.544	1.03
7500	340.0	3.60	3.57	12.50	3	2.424	0.94
9900	339.0	3.26	3.23	12.50	3	-	1.39
11100	338.0	3.36	3.32	12.50	3	2.385	1.14
0	336.5	2.21	2.10	10	3	-	0
9000	339.0	3.90	3.71	10	3	-	0.48
10200	340.0	4.01	3.80	10	3	-	0.44
11400	340.0	4.05	3.81	10	3	2.642	0.42
12600	340.0	3.83	3.55	10	3	2.841	0.56
13800	340.5	3.83	3.50	10	3	2.998	0.58
15000	340.5	3.83	3.44	10	3	3.128	0.58
16200	340.5	3.83	3.42	10	3	3.171	0.58
0	334.5	2.24	1.85	10	3	-	0
9300	340.0	4.44	3.60	10	3	-	0.26
10200	340.5	4.95	4.02	10	3	-	0.15
11100	340.0	4.87	3.84	10	3	3.685	0.15
12000	341.0	4.80	3.61	10	3	4.145	0.18

As far as the velocity v_x accepts a value close to v on the outer boundary of the hydrodynamic boundary layer of the thickness δ_h , it may be written:

$$\delta/\delta_h \sim v_x/v, \quad (2)$$

where δ is the distance from the crystal ace, m.

For the hydrodynamic boundary layer the following relationship may be used

$$\Phi d_k / \delta_h \approx \sqrt{Re_0}, \quad (3)$$

where Re_0 the local-oscillatory Reynolds number is defined by

$$Re_0 = 4Af\Phi d_k \rho / \mu, \quad (4)$$

where the mean velocity of oscillatory motion of the disk or the crystal $w = 4Af$; ρ is the fluid density, kg/m³; μ is the dynamic viscosity, Pa·s.

Taking into account the small value of Re_0 during the saccharose crystallization experiments ($Re_0 < 2$), it can be concluded from Eq.(3) that the hydrodynamic boundary layer δ_h is commensurable with the size of crystals and that the solution flow around crystals is laminar.

Substituting Eq.(2) into Eq.(1) and noticing that the first two terms in Eq.(1) are of the same order, one has

$$(v / \Phi d_k) \cdot \delta / \delta_h \approx D [1 / (\Phi d_k)^2 + 1 / \delta_d^2]. \quad (5)$$

It can be found from Eq.(3) that

$$v \approx \Phi d_k \nu / \delta_h^2, \quad (6)$$

where ν is the kinematic viscosity, m²/s.

Substituting Eq.(6) into Eq.(5), after simple transformations, noticing that $\delta = \delta_d$ on the boundary of a diffusional layer, one obtains

$$(\delta_d / \delta_h)^3 \approx 1 / Sc \cdot [(\delta_d / \Phi d_k)^2 + 1], \quad (7)$$

where Sc is the Schmidt number ($Sc = \mu / \rho D$).

Because of the awkwardness of the analytical solution of Eq.(7), it was solved by use of numerical procedures. Previously for solving this equation, the estimation of hydrodynamic boundary-layer δ_h thickness was found using Eq.(3).

The results of calculation of the hydrodynamic and diffusional boundary layer thickness for the mean crystal size $d = 6 \cdot 10^{-4}$ m in the experiments on isobaric crystallization of high- and low-grade sugar massecuites are shown in Table 2. Analyzing the results it can be concluded that δ_h and δ_d become tens of times as little during the realization of oscillatory crystal motion in comparison to their gravitational precipitation in massecuites /5/. It shows the expediency of developing the methods of acoustic effect on the sugar massecuites. For example, the method of oscillatory-state milk-sugar syrup boiling /6/, sugar massecuites /7/ has been developed, and a new type of apparatus for crystallization with vibrostirring mechanisms has been designed.

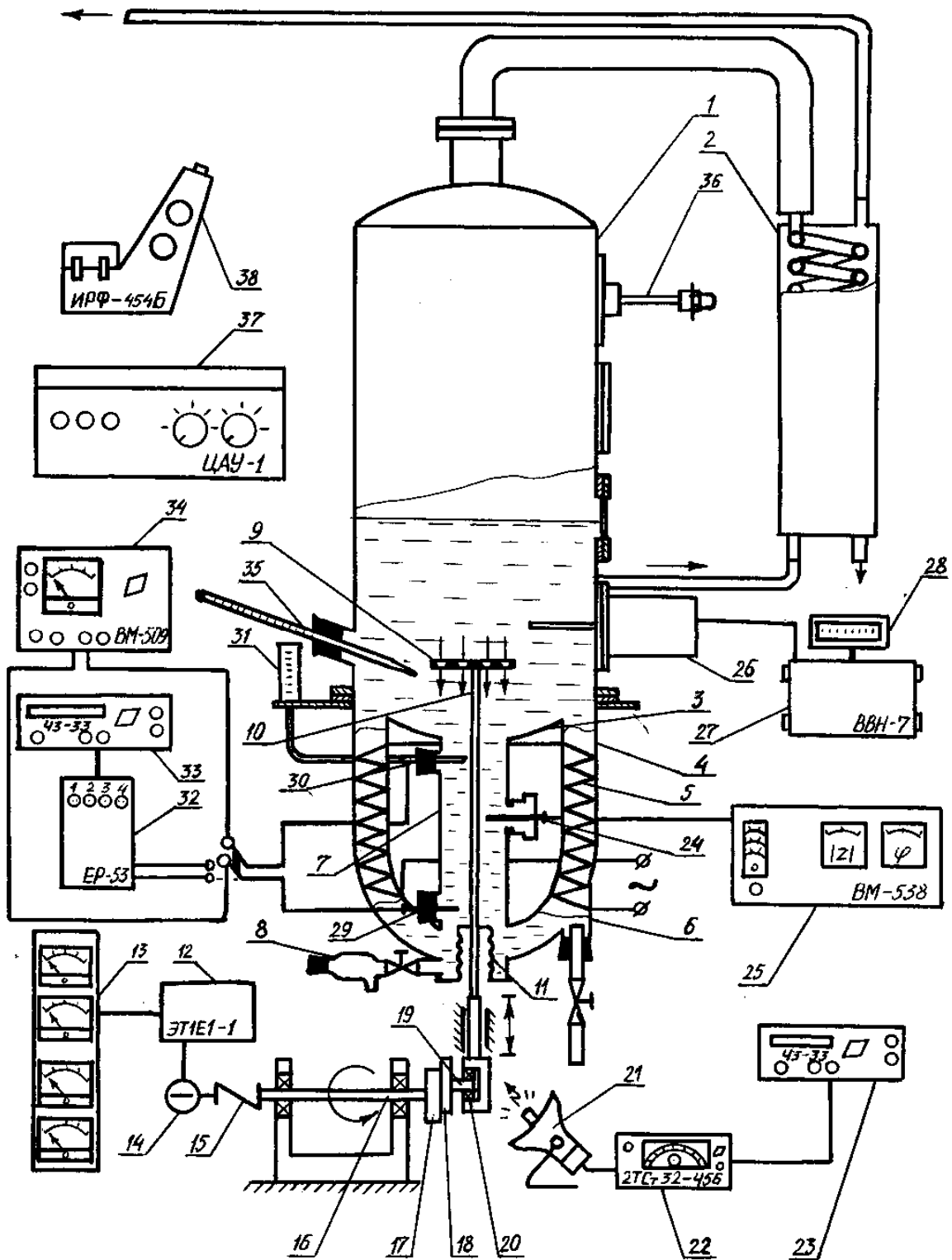


Fig. 1. Laboratory batch vacuum-pan crystallization apparatus with a vibrostirring mechanism

Table-2. Dependence of thickness of hydrodynamic and diffusional boundary layers on the Reynolds and Schmidt numbers during high- and low-grade sugar-masseccuite boiling

Sample number	Purity, %	$J \cdot 10^3, \text{m}^2 \cdot \text{s}^{-3}$	Re_0	$Sc \cdot 10^{-3}$	δ_h, mm	δ_d, mm
1	99.6	18	1.31	317.3	0.369	0.00541
2	99.6	18	1.10	399.2	0.403	0.00547
3	99.6	18	0.93	516.2	0.438	0.00546
4	99.6	18	0.87	574.3	0.453	0.00545
5	99.6	18	0.97	427.4	0.429	0.00570
6	99.6	18	1.03	378.4	0.416	0.00575
7	99.6	18	0.94	436.6	0.436	0.00574
8	99.6	18	1.39	284.8	0.358	0.00544
9	99.6	18	1.14	365.3	0.396	0.00553
10	95.1	9	0.48	663.3	0.610	0.00699
11	95.1	9	0.44	690.6	0.637	0.00720
12	95.1	9	0.42	715.3	0.652	0.00729
13	95.1	9	0.56	502.4	0.564	0.00710
14	95.1	9	0.58	460.5	0.555	0.00719
15	95.1	9	0.58	441.4	0.555	0.00729
16	95.1	9	0.58	435.4	0.555	0.00732
17	82.5	9	0.26	852.2	0.828	0.00874
18	82.5	9	0.15	1550.3	1.091	0.00943
19	82.5	9	0.15	1399.0	1.091	0.00975
20	82.5	9	0.18	1059.7	0.996	0.00977

REFERENCES

1. Ganiev R.F. et al. *Kolebatel'nye Yavleniya v Mnogofaznykh i ikh Ispoi'zovanie v Tekhnologii*. Kiev: Tekhnika, 1980.
2. Novickii B.G. *Primenenie Akusticheskikh Kolebanii v Khimiko-Technologicheskikh Processakh*. Moscow: Khimiya, 1983.
3. Petrov S.M. et al. *Vestnik of the Russian Academy Agricultural Science*. 1994. No. 5. P.56-59.
4. Sheptun V.M. et al. *Teor. Osnovy Khim. Tekhnol.* 1980. Vol. 14. No. 5. P.764-767.
5. Shestov A.G., Petrov S.M. *Izv. VUZov. Pishch. Tekhnologiya*. 1995. No. 3/4. P.46-48.
6. Russ. Patent 2048526, Int. Cl. C 13 K 5/00.
7. Russ. Patent 2155815, Int. Cl. C 13 F 1/02.