THE REGULARIZATION OF THE PARETO OPTIMAL SET FOR MALTICRITERIA OPTIMIZATION OF THE PROPERTIES OF MATERIALS.

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Abstract The methods of approximation of the feasible solution and Pareto optimal sets are considered in the paper. The approximation of the feasible solution set and Pareto optimal set, and the regularization of the Pareto optimal set are described.

Keywords The feasible solution set, the regularization, the Pareto optimal set, the approximation.

Let us consider an object whose operation is described by a system of equations (differential, algebraic, etc.) or whose performance criteria may be directly calculated. We assume that the system depends on r design variables $\alpha_1,...,\alpha_r$ representing a point $\alpha = (\alpha_1,...,\alpha_r)$ of an r-dimensional space . There also exist particular performance criteria, such as productivity, materials consumption, and efficiency. It is desired that, with other things being equal, these criteria, denoted by $\Phi_\nu(\alpha)$, $\nu = 1,...,k$ would have the extreme values. For simplicity, we assume that $\Phi_\nu(\alpha)$, are to be minimized.

Let $\Phi(\alpha) = (\Phi_1(\alpha),\ldots,\Phi_k(\alpha))$ is the criterion vector and D is the feasible set [1].

Definition1. A point $\alpha^0 \in D$, is called the Pareto optimal point if there exists no point $\alpha \in D$ such that $\Phi_\nu(\alpha) \le \Phi_\nu(\alpha^0)$ for all all $\nu = 1,\ldots,k$ and $\Phi_{\nu_0}(\alpha) < \Phi_{\nu_0}(\alpha^0)$ for at least one $\nu_0 \in \{1,\ldots,k\}$.

A set $P \subset D$ is called the Pareto optimal set if it consists of Pareto optimal points.

When solving the problem, one has to determine a design variable vector point $\alpha^0 \in P$, which is most preferable among the vectors belonging to set P.Let ε_ν be an admissible (in the expert's opinion) error in criterion Φ_ν. By ε we denote the error set $\{\varepsilon_\nu\}$, $\nu = 1,...,k$. We will say that region $\Phi(D)$ is approximated by a finite set $\Phi(D_\varepsilon)$ with an accuracy up to the set ε, if for any vector $\alpha \in D$, there can be found a vector $\beta \in D_\varepsilon$ such that $|\Phi_\nu(\alpha) - \Phi_\nu(\beta)| \le \varepsilon_\nu$, $\nu = 1,...,k$.

Let N_1 be the subset of the points of D that are either the Pareto optimal points or lie within the ε-neighborhood of a Pareto optimal point with respect to at least one criterion. In other words, $\Phi_\nu(\alpha^0) \le \Phi_\nu(\alpha) \le \Phi_\nu(\alpha^0) + \varepsilon_\nu$, where $\alpha^0 \in P$, and P is the Pareto optimal set. Also, let $N_2 = D \backslash N_1$ and $\bar{\varepsilon}_\nu > \varepsilon_\nu$, .

Definition 2. A feasible solution set $\Phi(D)$ is said to be normally approximated if any point of set N_1 is approximated to within an accuracy of ε, and any point of set N_2 to within an accuracy of $\bar{\varepsilon}$.

Theorem 1. If criteria $\Phi_\nu(\alpha)$ are continuous and satisfy the Lipschitz condition [1] then there exists a normal approximation $\Phi(D_\varepsilon)$ of a feasible solution set $\Phi(D)$,

Let P be the Pareto optimal set in the design variable space; $\Phi(P)$ be its image; and ε be a set of admissible errors. It is desirable to construct a finite Pareto optimal set $\Phi(P_\varepsilon)$ approximating $\Phi(P)$ to within an accuracy of ε .Let $\Phi(D_\varepsilon)$ be the ε-approximation of $\Phi(D)$, and P_ε be the Pareto optimal subset in D_ε. As has already been mentioned, the complexity of constructing a finite approximation of the Pareto optimal set results from the fact that, in general, in approximating the feasible solution set $\Phi(D)$ by a finite set $\Phi(D_\varepsilon)$ to within an accuracy of ε, one cannot achieve the approximation of $\Phi(P)$ with the same accuracy. Such problems are said to be ill-posed in the sense of Tikhonov[1]. Let us set $X=\{\Phi(D_\varepsilon), \Phi(D)\}$; $Y=\{\Phi(P_\varepsilon), \Phi(P)\}$,where $\varepsilon \to 0$. In spaces X and Y, the topology, that corresponds to the system of preferences on $\Phi(D)$ is specified [1]. In Theorem 2, we have to construct a Pareto optimal set $\Phi(P_\varepsilon)$ in which for any point $\Phi(\alpha^0) \in \Phi(P)$ and any of its ε-neighborhoods V_ε there may be found a point $\Phi(\beta) \in \Phi(P_\varepsilon)$ belonging to V_ε . Conversely, in the ε-neighborhood of any point $\Phi(\beta) \in \Phi(P_\varepsilon)$, there must exist a point $\Phi(\alpha^0) \in \Phi(P)$. The set $\Phi(P_\varepsilon)$ is called an approximation possessing property *M*. Let $\Phi(D_\varepsilon)$, an approximation of $\Phi(D)$, have been constructed.

Theorem 2. If the conditions of Theorem 1 are satisfied, then there exists an approximation $\Phi(P_\varepsilon)$ of Pareto set $\Phi(P)$ possessing the *M*-property.

This theorem solves the problem of the ill-posedness (in the sense of Tikhonov) of the Pareto optimal set approximation.

REFERENCES

[1] Statnikov, R.B., and J.B. Matusov, *Multicriteria Analysis in Engineering,* Dordrecht/Boston/London: Kluwer Academic Publishers, 2002.