ON A DECISION MODEL TO GRANT LOANS TO ENTERPRISES ON THE BASIS OF MARKOV MODELS FOR FINITE HORIZON

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Abstract*:* In this paper we propose a method to construct a quantitative assessment of decision-making based on a Markov model of decision-making about the possibility of granting loan to the borrowing enterprises from the lender (bank). As a results of the computational experiments for finite planning horizon and the principle of optimizing the income of the enterprise, it allows the lender (creditor) organization to make better informed decisions which is to maximize the expected income in the provision of the investigated credit enterprise. This technique is clearly justified in the application of the theory of decision-making and optimization theory. Based on the proposed method we can construct other methods of decision-making in assessing the creditworthiness of the enterprise in question.

Keywords*:* Decision, Markov models, the planning horizon, the transition probability matrix, the optimal expected revenue

Meeting the challenges of constructing models of decision-making remains in the currently weak-formalized area in which the quality of the solutions essentially depends on the experience and intuition of researchers [3]. Currently, there are many different methods of decision making, suitable for solving problems, and at the same time there are practically no formal recommendations on the selection method for a given task. In this context, the aim of this research is to develope a mathematical model based on the Markov model of decision-making.

Businesses wishing to get a loan from the credit institutions (banks), as at the time of the loan may be in one of three states: state 1 - prosperous company, state 2 - the financial condition of the company is such that it is in the "5 years before bankruptcy " state 3 -" a year before the bankruptcy ". It is known from [1] that a company may be in any of this state, if at least three indicators pointing to Beaver belong to this state.

We assume that the decision maker (DM) from the lending institutions (banks) consider two possible options for action (strategy): X_1- to give, X_2 - not to give the company credit. We also assume that, depending on the state of the company $S_j(t)$, $j = 1,2,3$, which is consistent, that its generated income can be calculated by the lending institutions (banks). For example, if at time t the company was in a state S_j and at time $(t+1)$ the state continues, it will be considered as the maximum income.

For example if $a_t = 5475$ thousnd rub, then the income matrix will be the limit as $R^t = \{r_{tj}\}$, where p_j^t is the matrix elements of transition probabilities P.

Assume that we know the statistical (accounting) information of a company for three years $t = 3$. On the basis of these data, we calculate the coefficients k_s, $s = 1, \cdots, 5$, from Beaver and probability [1] p_j^t, we have that the enterprise under investigationis is in $S_j(t)$ state at time $t = 1,2,3$, $\mathrm{j} = 1, 2, 3$. For illustrative purposes, the stitistical data for $t = 1,2,3$ years was taken fro a particilar company ОАО «Ленмолоко» [3]. The transition probability matrix is as follows P:

$$P = \begin{pmatrix} p_1^1 & p_1^2 & p_1^3 \\ p_2^1 & p_2^2 & p_2^3 \\ p_3^1 & p_3^2 & p_3^3 \end{pmatrix}; (1)$$

Elements p_j^t of the matrix denotes transition probability of the system in state $S_j(t)$. Thus the rows of the matrix correspond to the "state S_j" and the colums - "time $t = 1,2,3$". The sum of elements of each row is unity that is $\sum_{t=1}^{3} P_j^t = 1$, $j = 1,2,3$.

For the simulation of the situation in the company, the transition probability matrix may be represented in the income matrix as follows: $R = \{r_j^t; r_j^t = a_t \cdot p_j^t\}$:

$$R = \begin{pmatrix} r_1^1 & r_1^2 & r_1^3 \\ r_2^1 & r_2^2 & r_2^3 \\ r_3^1 & r_3^2 & r_3^3 \end{pmatrix},$$

element r_{tj} of the matrix denotes the income obtained at time t when the system is in state j.

With the matrix P and R one can simply predict the results of the system. In this case, the set of feasible solutions $G = \{X_1, X_2\}$, where X_1 - the decision to issue a loan the company, and X_2 - not to issue. Thus, the transition probability matrix is given by [4,5]:

$$P(i|X_{i-1}) = \begin{cases} P_1, & X_{i-1} = X_1, \\ P_2, & X_{i-1} = X_2. \end{cases}$$

The transition of the company from one state to another is associated revenuematrix $R(i|X_i) = (r_j^t(i|X_i))$ whose element $r_j^t(i|X_i)$ is the income (positive values) for the t-th stage. At the same time, the income in t-th stage is associated with the transition of the company from state $S_{(j-1)}$, in which it was after the $(t-1)$-th stage, a state S_j when deciding whether $X_i \in G$.

Values [4,5] $v_t\left(X_{i-1}\right)=\sum_{tj=1}^{3} p_j^t\left(i \middle| X_{i-1}\right) r_j^t\left(i \middle| X_{i-1}\right)$ defines the expected revenue for the t-th stage, if after the $(t-1)$-th stage the company was able to be in S_j state and it was decided to $X_{i-1} \in G$.

It is necessary to note that the decision-maker may be interested in the value of the expected income at a predetermined strategy of behavior in case of a state company S_j. For example, PMD may consider that after $(t-1)$-th stage if the enterprise is in State S_j, regardless of the particular value i, it is necessary to make a decision $X_{i-1} \in G$.

For a finite planning horizon, ie, a finite number of steps $(t<\infty)$ of Markov decision problem with the principle of optimality, which is to maximize the expected revenue for the t stages, it may be represented as a dynamic programming problem.

Let $f_t(j)$ - expected optimal revenue (that is optimality principle in its best use) for stages with numbers $t,(t+1),\cdots,N$, provided that after the t-th stage of the test, the company is able to be state S_j, $j=1,2,3$. Since the planning horizon is finite, then the optimal expected revenue requirements must be satisfied

$$f_{N+1}(j) \equiv 0, \; j=1,2,3$$

Expected optimal revenue $f_t(j)$ phases numbered $t,(t+1),\cdots,N$ consists of two component The first component - optimal revenue for $(t+1)$-m stage, due to a transition of the company from a state S_j in which it was at the t-th stage, in any admissible state, $j=\overline{1,m}$ [2,5]. $\max\limits_{X_i \in G} v_t\left(X_i\right)$; $v_t\left(X_i\right)=\sum_{j=1}^{m} p_j^t\left(i+1 \middle| X_i\right) r_j^t\left(i+1 \middle| X_i\right)$ (2), where $p_j^t\left(i+1 \middle| X_i\right)$ - the conditional probability that, after the $(t+1)$-th stage the company will be in a state S_j and has a feasible solution X_i; $r_j^t\left(i+1 \middle| X_i\right)$- . income of the company in state S_j, in which it was after the t-th stage as a result of X_i, from the set of feasible solutions G. The second component of the optimal revenue $f_i(j)$ determined by a combination of optimal income $f_{t+1}(t)$, $t=\overline{1,m}$, with transition probabilities $p_j^t\left(i+1 \middle| X_i\right)$, $k=\overline{1,m}$:

$$\max_{X_i \in G} \sum_{t=1}^{3} p_j^t\left(i+1 \middle| X_i\right) f_{i+1}(t).$$

We come to the recurrence equation of dynamic programming with a finite number of steps linking the expected optimal revenue $f_i(j)$, $j=\overline{1,m}$ и $f_{i+1}(t)$, $t=\overline{1,m}$, $m=3$:

$$f_t(j)=\max_{X_i\in G}\left\{v_t(X_i)+\sum_{t=1}^{m}p_j^t(i+1|X_i)\,f_{i+1}(t)\right\},\ i=\overline{1,N-1},\ j=\overline{1,m}.$$

In this case, we recall that $f_{N+1}(t)\equiv 0$, $t=\overline{1,m}$ и $v_t(X_i)=\sum_{t=1}^{m}p_j^t(i+1|X_i)r_j^t(i+1|X_i)$, $j=\overline{1,m}$

Example: Assume that the lending institutions (Bank), considers the enterprise in question to provide credit. To do this, the bank needs to develop an optimal behavioral strategy, that is, to maximize the total return for the loan to the entity. Recall that the investigated company has identified three possible S states: State 1 - prosperous company, state 2 - the financial condition of the company is such that it is in the "5 years before bankruptcy," state 3 - "a year before the bankruptcy." Set of feasible solutions $G=\{X_1,X_2\}$, where X_1 - the decision to issue a loan company, and X_2 - not to give. Transition probability matrix has the following form (see. (1)) [5].

This discourse was taken from a known statical (accounting) data on a gravity $n=3$. On the basis of these data, we calculate the coefficients W. Beaver k_i, $i=1,\cdots,5$ and probability p_j^t that the investigated the company is in $S_j(t)$ state at time t, $t=1,\cdots,3$, $j=1,2,3$. Then, the transition probability matrix P_1 is as follows: $P_1=\begin{pmatrix}0{,}6 & 0 & 0{,}4\\ 0{,}6 & 0{,}4 & 0\\ 0{,}4 & 0{,}2 & 0{,}4\end{pmatrix}$. The sum of elements in each row is unity, that is $\sum_{t=1}^{3}P_j^t=1$, $j=1,2,3$.

Suppose that on the basis of the resulting matrix of transition probabilities matrix P_1 is calculated as follows: $P_2=P_1P_1$: $P_2=\begin{pmatrix}0{,}04 & 0{,}35 & 0{,}51\\ 0 & 0{,}25 & 0{,}75\\ 0 & 0 & 1\end{pmatrix}$, and the income matrix R in accordance with the transition probability matrix is calculated by the following formula $r_j^t=a_t\cdot p_j^t$ where $a_t=5475$ th. rub. - desired income at time t, and p_j^t - elements of the matrix of transition probabilities

$$R_1=\begin{pmatrix}3285 & 0 & 2190\\ 3285 & 2190 & 0\\ 2190 & 1095 & 2190\end{pmatrix};\ R_2=\begin{pmatrix}219 & 1916{,}25 & 3339{,}5\\ 0 & 1368{,}75 & 4106{,}25\\ 0 & 0 & 3475\end{pmatrix}$$

We assume that the planning horizon is $N=3$. From these matrices P_1, P_2, R_1, R_2, we calculate the expected revenues from the formula (2), due to the transition from one state to another in different variants of feasible solutions of the sets G of the companies considered:

$v_1(X_1) = 0{,}6 \times 3285 + 0 \times 0 + 0{,}4 \times 2190 = 2847$,

$v_2(X_1) = 0{,}6 \times 3285 + 0{,}4 \times 2190 + 0 \times 0 = 2847$,

$v_3(X_1) = 0{,}4 \times 2190 + 0{,}2 \times 1095 + 0{,}4 \times 2190 = 1971$,

$v_1(X_2) = 0{,}04 \times 219 + 0{,}35 \times 1916{,}25 + 0{,}51 \times 3339{,}5 = 2716{,}695$,

$v_2(X_2) = 0 \times 0 + 0{,}25 \times 1368{,}75 + 0{,}75 \times 4106{,}25 = 3121{,}775$,

$v_3(X_2) = 0 \times 0 + 0 \times 0 + 1 \times 3475 = 3475$.

For clarity, we use the tabular algorithm for solving this problem.

Table 1. Calculations of optimal expected revenue for the 1st stage of planning.

j	$v_j(X_i)$		Optimal expected revenue, $f_3(j)$	Optimal decision
	$i=1$	$i=2$		
1.	2847	2716,695	2847	X_1
2.	2847	3121,775	3121,775	X_2
3.	1971	3447	3447	X_2

Table 2. Calculation of the optimal expected revenue for the second stage of planning.

j	$v_j(X_i) + \sum_{t=1}^{3} p_j^t(i+1 \vert X_i) f_3(j)$		Optimal expected revenue, $f_2(j)$	Optimal decision
	$i=1$	$i=2$		
1.	$2847 + 0{,}6 \times 2847 + {}$ $+ 0 \times 3121{,}775 + 0{,}4 \times 3447 = 5934$	$2716{,}695 + 0{,}04 \times 2847 + {}$ $+ 0{,}65 \times 3121{,}775 + {}$ $+ 0{,}51 \times 3447 = 4171{,}698$	5934	X_1
2.	$2847 + 0{,}6 \times 2847 + {}$ $+ 0{,}4 \times 3121{,}775 + {}$ $+ 0 \times 3447 = 5803{,}91$	$3121{,}775 + 0 \times 2847 + {}$ $+ 0{,}25 \times 3121{,}775 + {}$ $+ 0{,}75 \times 3447 = 6487{,}466$	6487,466	X_2
3.	$1971 + 0{,}4 \times 2847 + {}$ $+ 0{,}2 \times 3121{,}775 + {}$ $+ 0{,}4 \times 3447 = 15512{,}95$	$3447 + 0 \times 2847 + {}$ $+ 0 \times 3121{,}775 + {}$ $+ 1 \times 3447 = 6894$	15512,95	X_1

Table 3. Calculation of the optimal expected revenue for the third stage of planning.

j	$v_j(X_i) + \sum_{t=1}^{3} p_j^t(i+1 \vert X_i) f_2(j)$		Optimal expected revenue, $f_1(j)$	Optimal decision
	$i=1$	$i=2$		
1.	$2847 + 0{,}6 \times 5934 + {}$ $+ 0 \times 6487{,}466 + {}$ $+ 0{,}4 \times 15512{,}95 = 12612{,}58$	$2716{,}695 + 0{,}04 \times 5934$ $+ 0{,}65 \times 6487{,}466 + {}$ $+ 0{,}51 \times 15512{,}95$ $= 15082{,}5124$	15082,5124	X_2

2.	$2847+0{,}6\times 5934+$ $+0{,}4\times 6487{,}446+$ $+0\times 15512{,}95=9002{,}378$	$3121{,}775+0\times 5934+$ $+0{,}25\times 6487{,}466+$ $+0{,}75\times 15512{,}95$ $=16378{,}354$	16378,354	X_2
3.	$1971+0{,}4\times 5934+$ $+0{,}2\times 648{,}466+$ $+0{,}4\times 15512{,}95=10697{,}4732$	$3447+0\times 5934+$ $+0\times 6487{,}466+$ $+1\times 15512{,}95$ $=18959{,}95$	18959,95	X_2

From the results, we conclude that the best solution for the three years (phase) is X_2 - not to give credit to the enterprise, regardless of the economic market situation. For all three years (stages) it is optimal to assume X_2 as the feasible solution for all the possible states of the enterprise.

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